Graphing Rational Functions

These notes are intended as a summary of section 2.4 (p. 129 – 133) in your workbook. You should also read the section for more complete explanations and additional examples.

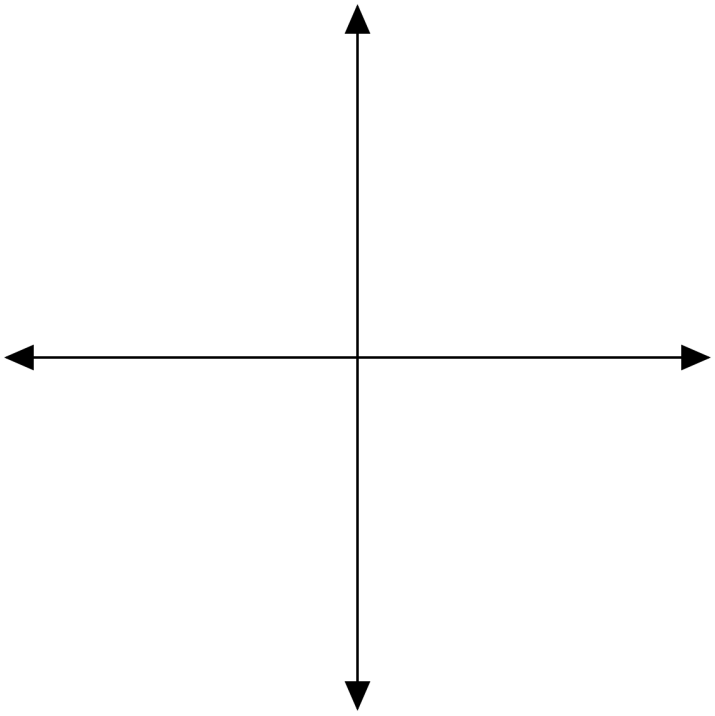
Graphing Rational Functions

To sketch the graph of a rational function:

1. Factor the numerator and denominator.
2. Find any non-permissible values of x.
    - common factor:
        - hole at NPV if factor's top degree $\geq$ bottom degree
        - vertical asymptote at NPV if factor's top degree < bottom degree
    - no common factor:
        - vertical asymptote at NPV
3. Find any horizontal or oblique asymptotes.
    - common factor:
        - no horizontal or oblique asymptotes
    - no common factor:
        - horizontal asymptote at $y = 0$ if function's top degree < bottom degree
        - horiizontal asymptote at $y = \frac{a}{b}$ if function's top degree = bottom degree
        - oblique asymptote if function's top degree = bottom degree + 1
4. Find x-intercepts, y-intercepts, and any other key points (at least one per section).
5. Draw the graph (asymptotic behavior).

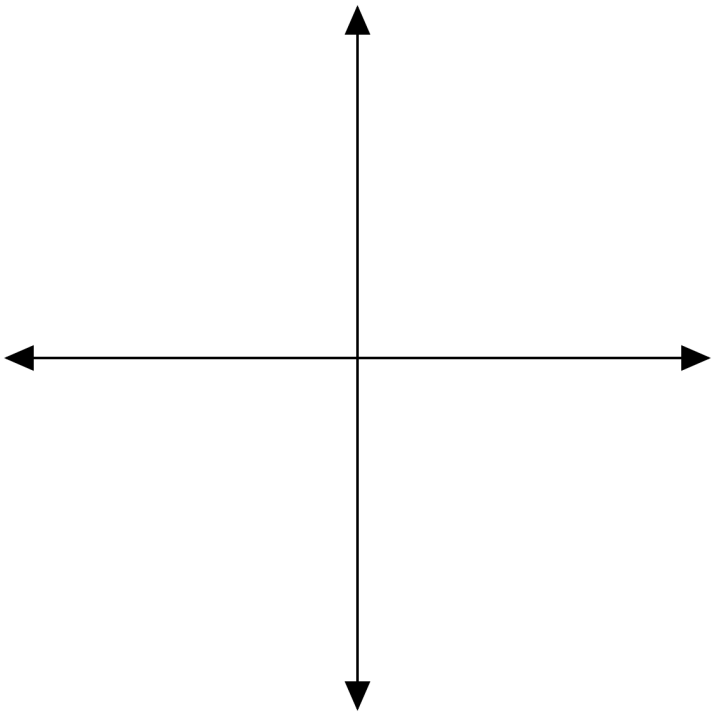
Example 1 (sidebar p. 130)
Sketch the graph of this rational function, then state the domain and range.

$$y = \frac{x^2 - 5x + 4}{1 - x}$$



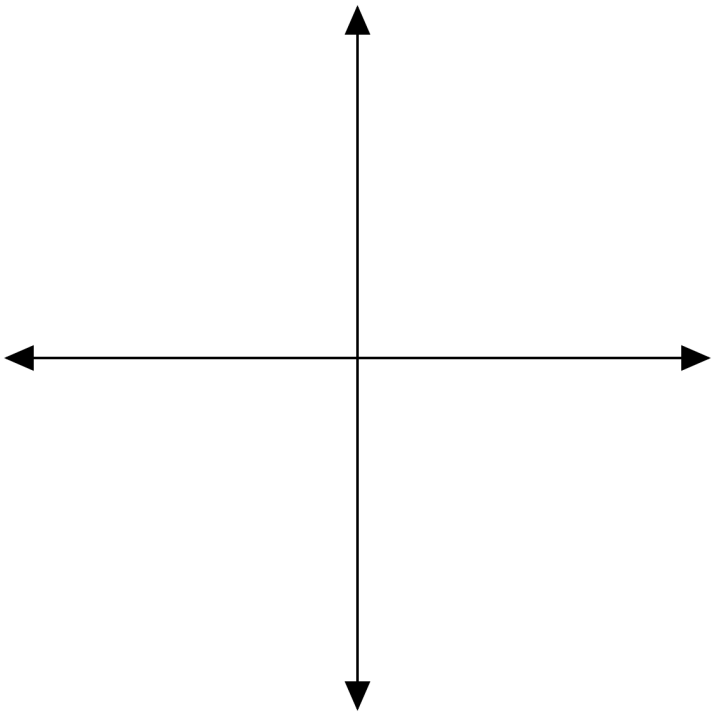
Example 2 (sidebar p. 131)
Sketch the graph of this rational function, then state the domain.

$$y = \frac{-2x^2}{x^2 - 25}$$



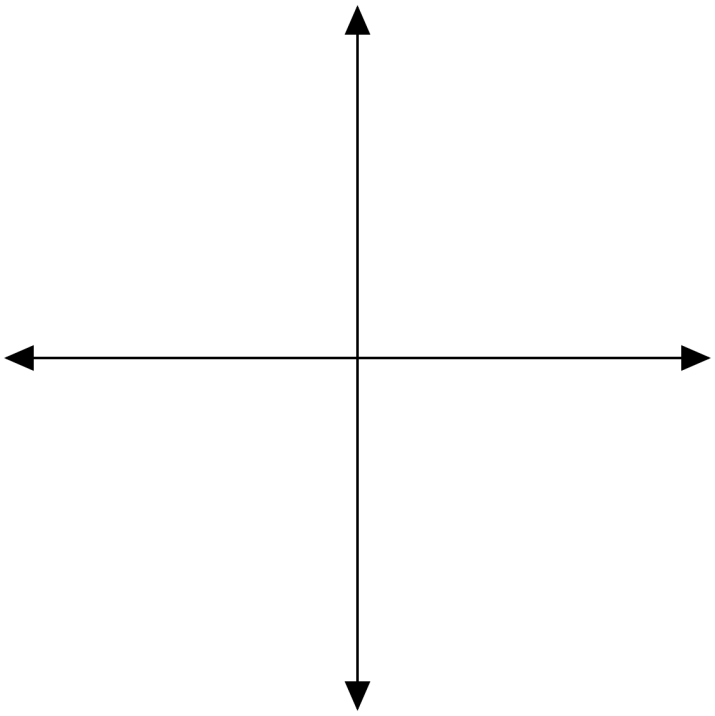
Example 3 (sidebar p. 132)
Sketch the graph of this rational function, then state the domain and range.

$$y = \frac{-2x^2 + 5x - 2}{x - 1}$$



Example 4 (sidebar p. 133)
Sketch the graph of this rational function, then state its domain and range.

$$y = \frac{x+3}{x^2-9}$$



Homework: #3 – 5 in the section 2.4 exercises (p. 134 – 141). Answers on p. 142.